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# Comparison of different methods for estimating muscle forces in human movement

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## Abstract

The aim of this study was to compare muscle-force estimates derived for human locomotion using three different methods commonly reported in the literature: static optimisation (SO), computed muscle control (CMC) and neuromusculoskeletal tracking (NMT). In contrast with SO, CMC and NMT calculate muscle forces dynamically by including muscle activation dynamics. Furthermore, NMT utilises a time-dependent performance criterion, wherein a single optimisation problem is solved over the entire time interval of the task. Each of these methods was used in conjunction with musculoskeletal modelling and experimental gait data to determine lower-limb muscle forces for self-selected speeds of walking and running. Correlation analyses were performed for each muscle to quantify differences between the various muscle-force solutions. The patterns of muscle loading predicted by the three methods were similar for both walking and running. The correlation coefficient between any two sets of muscle-force solutions ranged from 0.46 to 0.99 ( $p < 0.001$  for all muscles). These results suggest that the robustness and efficiency of static optimisation make it the most attractive method for estimating muscle forces in human locomotion.

## Keywords

Inverse dynamics, forward dynamics, joint torque, gait biomechanics, walking, running, motion simulation

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## Introduction

Accurate knowledge of muscle forces is essential for characterising muscle function and for developing new methods for treating patients with movement disorders.<sup>1</sup> Because direct measurement of muscle forces in vivo is not possible, joint kinematic and ground reaction force data from gait-analysis experiments are often used in conjunction with musculoskeletal modelling to predict muscle forces non-invasively.<sup>2,3</sup>

One of the main challenges in applying computational modelling is the muscle-moment redundancy problem.<sup>4</sup> Because each joint is spanned by several muscles, a net joint moment can be produced by an infinite number of muscle recruitment solutions. Inverse and forward-dynamics techniques have been widely used to solve this indeterminate problem.<sup>2,3</sup> The inverse-dynamics method uses the experimental joint kinematics and ground reaction force data as inputs to a musculoskeletal model to calculate the net joint moments applied about each joint. The muscle-moment redundancy problem is then solved at each time instant using static

optimisation to minimise a given performance criterion (e.g. sum of squares of muscle activations<sup>2,5,6</sup>). While static optimisation is computationally efficient, it is not designed to incorporate time-dependent muscle properties such as the time delay in the transformation of neural excitation to muscle activation (i.e. muscle activation dynamics), or time-dependent performance criteria such as minimum muscular effort<sup>7-9</sup> or minimum metabolic energy consumption.<sup>10</sup>

In contrast, the forward-dynamics method uses neural excitation signals as inputs to a model of the neuromusculoskeletal system. The equations representing muscle activation dynamics, muscle contraction dynamics and body-segmental dynamics are integrated

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simultaneously to predict the resulting joint motion. Dynamic optimisation or optimal control has been used to predict joint motion and ground reaction forces in walking by minimising the muscle metabolic energy consumed over a full stride cycle.<sup>10,11</sup> Unfortunately, the vast computation time needed to converge to a solution makes this approach practically infeasible.<sup>10, 11</sup> Computed muscle control (CMC)<sup>12,13</sup> and neuromusculoskeletal tracking (NMT)<sup>8</sup> are two recent approaches designed for generating forward-dynamics simulations of movement more efficiently. Both methods use feedback control theory to generate a stable simulation while including muscle activation dynamics to account for the time delay in muscle-force development. Although CMC and NMT are conceptually similar, they differ in the approach used to solve the muscle-moment redundancy problem. Whereas the CMC method uses static optimisation to resolve the muscle redundancy problem at each instant along the movement trajectory (see Figure 1 in Thelen and Anderson<sup>12</sup>), NMT solves the same problem dynamically by minimising a time-dependent performance criterion over the entire period of the task (see equation (11) in Seth and Pandya<sup>8</sup>).

Anderson and Pandya<sup>6</sup> quantitatively compared lower-limb muscle forces obtained from static and dynamic optimisation solutions of normal walking and found no significant differences between these two approaches. They concluded that static optimisation provides reasonable predictions of muscle forces when accurate joint moments are available, and suggested that the use of the more time-consuming dynamic optimisation approach is less justified. This conclusion,

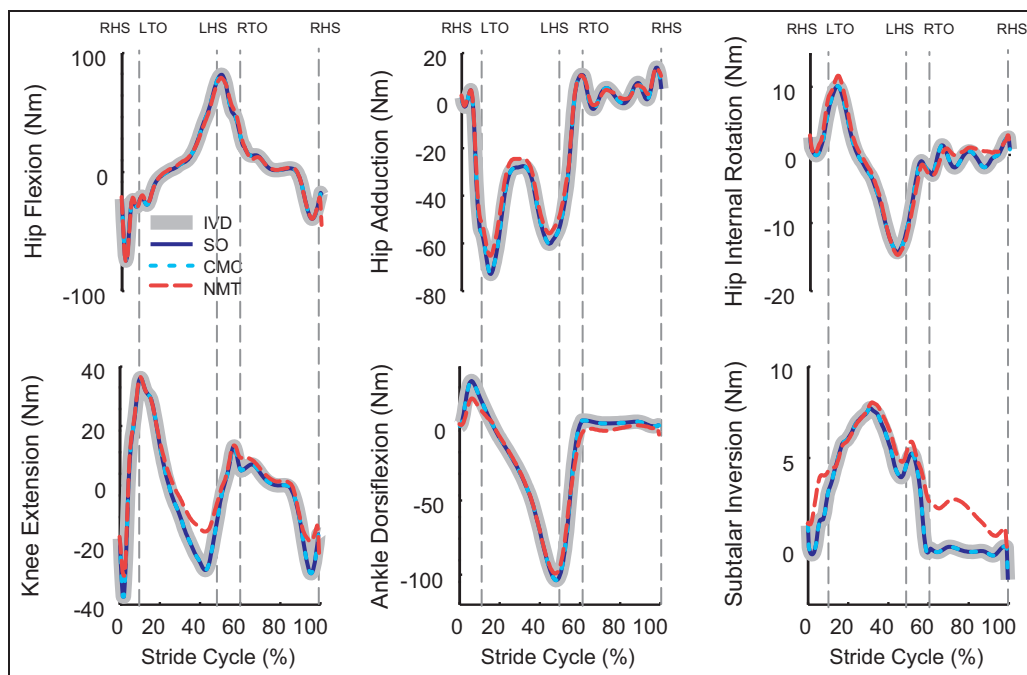
however, was based on simulated gait data rather than actual gait measurements. It is also unclear whether this finding applies to motor tasks characterised by more rapid joint movements such as running. Although a number of studies have used both inverse and forward-dynamics methods to study muscle function during running,<sup>14-18</sup> none of these studies have conducted a quantitative comparison of the muscle-force solutions obtained from these two methods.

The overall goal of the present study was to compare muscle-force estimates derived for human locomotion using three different methods commonly reported in the literature: static optimisation (SO), CMC and NMT. Muscle-actuated simulations of walking and running were generated for a single subject by combining musculoskeletal modelling and biomechanical gait experiments. In contrast to the study by Anderson and Pandya,<sup>10</sup> muscle forces were calculated using measurements of joint motion and ground reaction forces as inputs to each method. The specific aim of this paper is to determine the extent to which inclusion of muscle activation dynamics and/or a time-dependent performance criterion influences predictions of lower-limb muscle forces for walking and running.

## Methods

### Gait experiments

Data were collected from one healthy female adult (age: 25 years; height: 177 cm; mass: 64 kg) in the Biomechanics Laboratory at the Australian Institute of Sport. The subject provided informed written consent



**Figure 1.** Comparison of net joint moments computed from inverse dynamics (IVD) and net joint moments resulting from three muscle-force solutions (SO, CMC and NMT) for walking at the preferred speed.

after approval was obtained from the relevant institutional ethics committees. Marker-derived kinematic data were acquired using a three-dimensional motion analysis system (Vicon, Oxford Metrics, Oxford, UK). Small reflective markers (14 mm) were mounted on the trunk and both lower limbs. Marker trajectories were recorded using 22 optical infrared cameras sampling at 250 Hz as the subject walked and ran at her preferred speeds (walking: 1.61 m/s; running: 3.48 m/s) along a synthetic track. A fourth-order Butterworth low-pass filter (4 Hz) was used to smooth the marker trajectories. Ground reaction forces were measured simultaneously using eight force plates (Kistler Instrument Corp., Amherst, New York, USA) arranged in series along the track. The force plate data were low-pass filtered with a fourth-order Butterworth filter (60 Hz) to remove high-frequency noise. Surface electrodes were placed on the subject's right leg to record electromyographic (EMG) activity from seven muscles: gluteus maximus, gluteus medius, medial hamstrings, vastus lateralis, rectus femoris, medial gastrocnemius and soleus.

### Inverse- and forward-dynamics methods

SO, CMC and NMT were each implemented separately in the calculation of lower-limb muscle forces (Table 1). SO decomposes the net joint moments into individual muscle forces by minimising a time-independent performance criterion at each time instant. In the present study, SO was implemented by minimising the sum of the squares of all muscle activations at each instant of the stride cycle, and the calculated value of each muscle force was subject to physiological constraints according to its force-length-velocity properties.<sup>6</sup>

CMC produces a forward simulation of the prescribed task by using a proportional-integral controller to track the joint angular accelerations measured from a gait experiment.<sup>12</sup> The required set of neural excitations is found by solving a static optimisation problem that minimises the sum of the squares of all muscle activations at each instant of the task. Time-dependent performance criteria are therefore not incorporated in the formulation of the CMC problem; however, the effects of muscle activation dynamics are taken into account by performing a forward integration of the system equations using muscle excitations as inputs to the model (Table 1).

NMT combines feedback linearisation with optimal control theory to track the net joint moments obtained

from an inverse-dynamics analysis.<sup>8</sup> Although NMT is similar to CMC in that both methods implement muscle activation dynamics to solve a tracking problem, the NMT method minimises the sum of the squares of all muscle activations and the sum of squares of the joint torque tracking errors over the entire time interval of the task. Thus, the NMT method requires two sets of predefined weightings (one for minimising joint-torque tracking errors, and another for minimising muscle activation) in its time-dependent performance criterion to balance between joint torque tracking and the minimisation of muscle activation, whereas the SO and CMC methods treat the minimisation of joint-torque tracking errors as an equality constraint in the formulation of the optimisation problem. The NMT method is therefore able to account for the effects of both muscle activation dynamics and a time-dependent performance criterion in estimating muscle forces during movement (Table 1).

### Musculoskeletal modelling

A 10-segment, 23-degree-of-freedom musculoskeletal model was used to determine lower-limb muscle forces for one complete stride cycle of both walking and running.<sup>19</sup> The head, arms and torso were modelled as a single rigid body, which articulated with the pelvis via a ball-and-socket joint. Each hip was modelled as a ball-and-socket joint, each knee as a hinge joint, each ankle-subtalar complex as a universal joint and each metatarsal joint as a hinge. A subject-specific model of the skeleton was generated by scaling the anthropometric properties of each segment according to the subject's height and weight. The model was actuated by 54 Hill-type muscle-tendon units.<sup>20</sup> The force-producing properties, attachment sites and paths of all muscle-tendon units used in the model were based on data reported by Anderson and Pandy.<sup>19</sup>

### Muscle-force calculations

Joint angles and net joint moments for walking and running were computed prior to calculation of lower-limb muscle forces. An inverse-kinematics analysis was performed by solving a weighted least-squares optimisation problem<sup>21</sup> to determine the joint angles in the model that most accurately reproduced the measured marker coordinates. A single set of optimal joint angles was then applied to the musculoskeletal model in conjunction with force plate data to compute the net moments exerted about the lower-limb joints.

Lower-limb muscle forces for both walking and running were determined by applying the SO, CMC and NMT algorithms to the musculoskeletal model. Eight major muscles were selected for comparison across the three sets of muscle-force solutions ( $\mathbf{F}^{\text{SO}}$ ,  $\mathbf{F}^{\text{CMC}}$  and  $\mathbf{F}^{\text{NMT}}$ ): SOL (soleus), GAS (medial and lateral portions of gastrocnemius combined), VAS (vastus medialis, vastus intermedius and vastus lateralis combined), RF

**Table 1.** Comparison of SO, CMC and NMT.

Algorithm	Muscle activation dynamics	Time-dependent performance criterion
SO	×	×
CMC	√	×
NMT	√	√

(rectus femoris), GMAX (gluteus maximus), GMED (anterior and posterior portions of gluteus medius combined), HAMS (medial and lateral portions of hamstrings combined) and ILPSO (iliacus and psoas combined). A correlation coefficient ( $R$ ) between any two of the solutions was calculated for each muscle. The significance ( $p$ -value) of each correlation was also computed.

## Results

CMC accurately reproduced the sagittal-plane net joint moments computed from inverse dynamics for both tasks with a root-mean-square (RMS) difference of less than 1 Nm, but larger variability was evident in the frontal and transverse-plane joint moments (Figures 1 and 2, compare CMC with IVD). For example, an RMS difference of 8 Nm was observed in the hip internal rotation moment generated for running (Figure 2). The NMT algorithm was also able to track the patterns of all net joint moments computed from inverse dynamics for both walking and running with an RMS difference of less than 15 Nm (Figures 1 and 2, compare NMT with IVD). Similar to the CMC results, greater variability was found in the hip internal rotation moment for running compared with the other joint moments (Figure 2). In contrast to the forward-dynamics techniques, SO successfully reproduced the net joint moments in all three planes computed from inverse dynamics for both walking and running with an RMS difference of less than 1 Nm (Figures 1 and 2, compare SO with IVD).

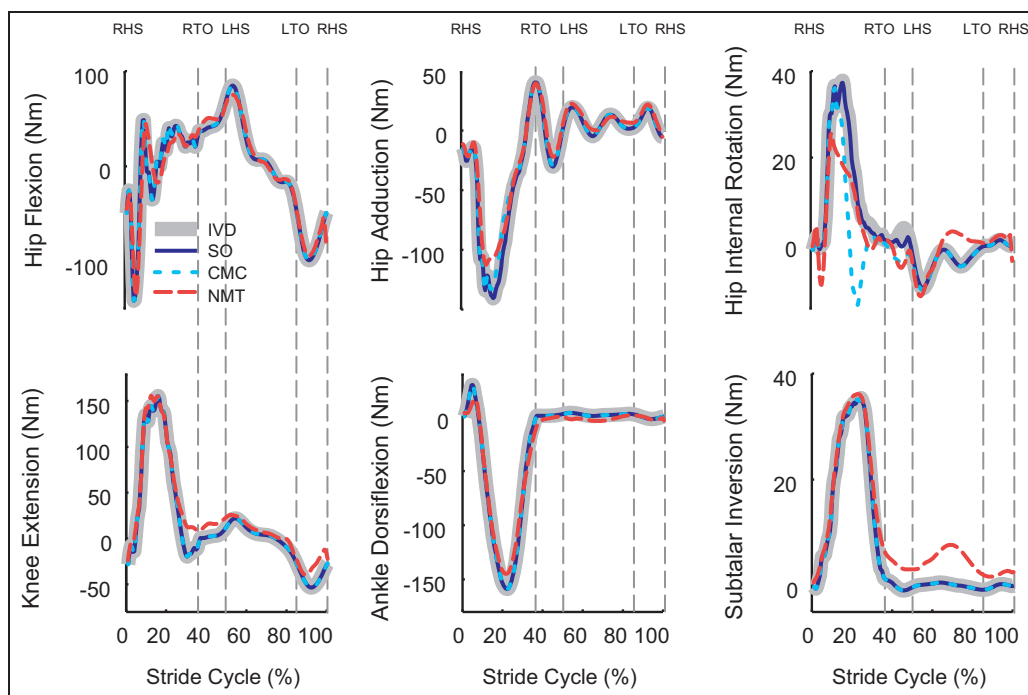
Muscle-force patterns predicted by all three methods for walking and running were consistent with the

sequence and timing of EMG measured for the subject (Figures 3 and 4). Muscle-force estimates derived from any two methods were similar for walking, with  $R$  values ranging from 0.46 to 0.99 (Figure 5). The CMC and NMT solutions for RF muscle force exhibited the least correlation with  $R = 0.46$ . The NMT solution showed that RF reached its peak value in the first half of the stance, whereas the CMC solution indicated that the peak force for this muscle occurred in the second half of the stance. For running, the correlation between any two muscle-force solutions was similar to that found in walking, with  $R$  values ranging from 0.51 to 0.99 (Figure 5). However, an increase in the speed of locomotion improved the correlation between the CMC and NMT solutions for RF force significantly, with  $R = 0.77$  obtained for this muscle in running. All correlations were significant with  $p < 0.001$ .

## Discussion

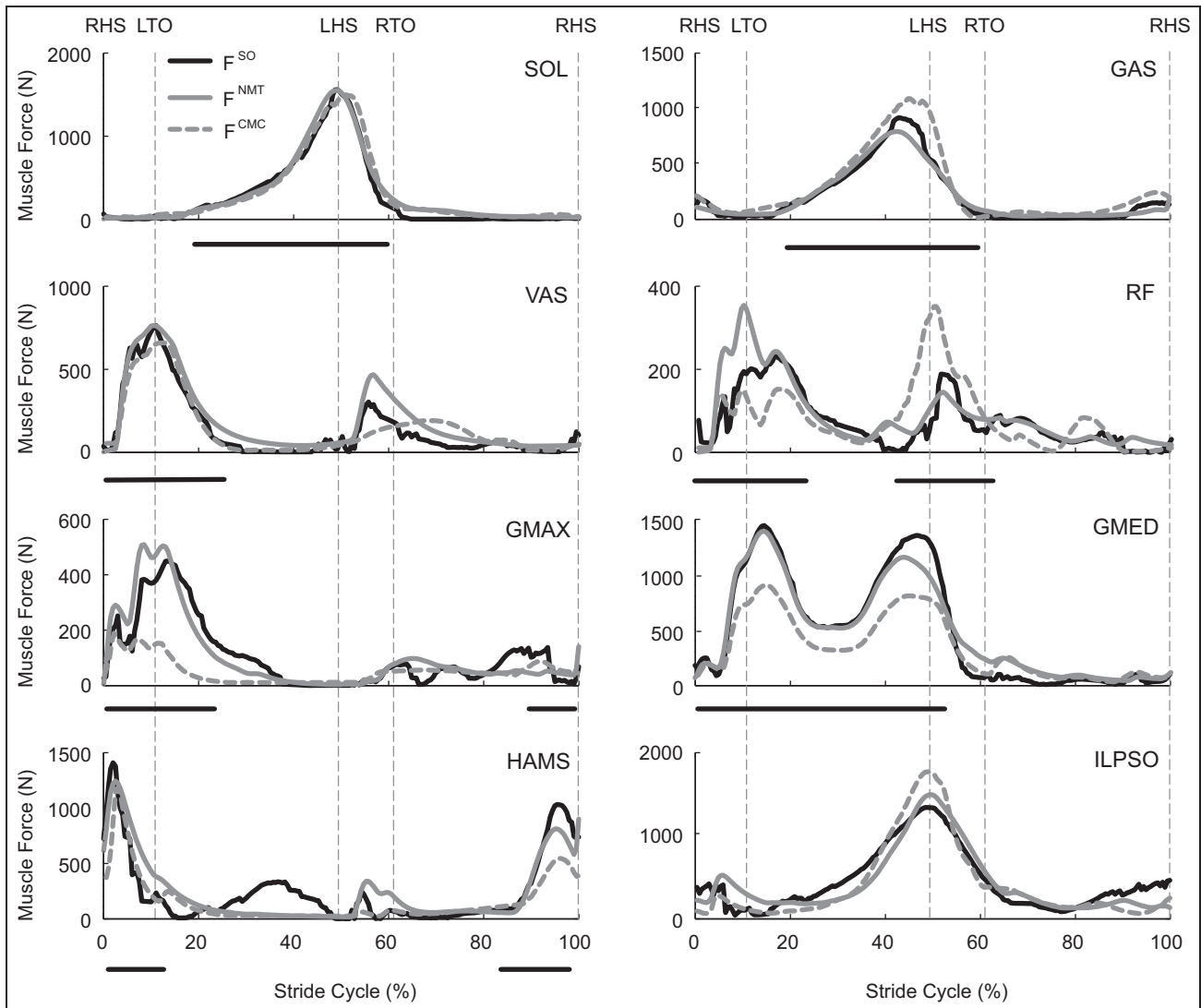
This study compared muscle-force estimates obtained for walking and running using three different methods commonly reported in the literature: SO, CMC and NMT. The patterns of muscle forces predicted by these methods were similar for both walking (Figure 3) and running (Figure 4), suggesting that muscle-force calculations are not significantly influenced by the inclusion of either muscle activation dynamics or a time-dependent performance criterion.

The current analysis was associated with a number of limitations. First, the results are based on data obtained from only one subject because the aim was to



**Figure 2.** Comparison of net joint moments computed from inverse dynamics (IVD) and net joint moments resulting from three muscle-force solutions (SO, CMC and NMT) for running at the preferred speed.



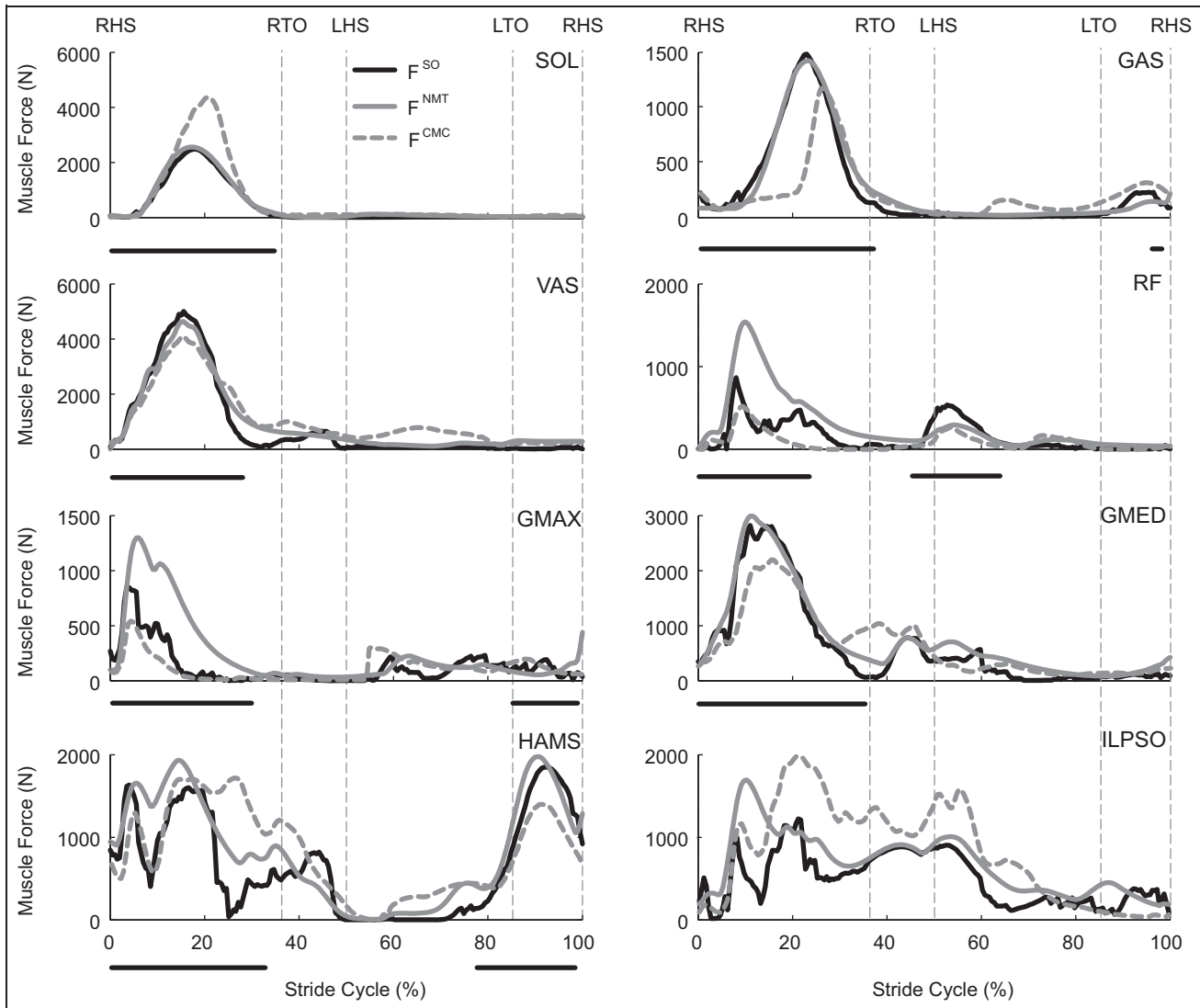


**Figure 3.** Comparison of three sets of muscle-force solutions ( $F^{SO}$ ,  $F^{CMC}$  and  $F^{NMT}$ ) for walking at the preferred speed. The horizontal bars indicate the periods of EMG activity recorded for the subject. No EMG data were recorded for ILPSO.

compare muscle-force estimates derived from different optimisation methods rather than study the functional roles of muscles during gait. Nonetheless, the joint kinematics and ground reaction forces measured for both walking and running were well within the normal ranges reported in the literature. Furthermore, the EMG data recorded for our subject were consistent with speed-matched EMG reported by others (e.g. compare EMG bars in Figures 3 and 4 with the results at '5 km/h' and '12 km/h' presented by Cappellini et al.<sup>22</sup> in their Figure 1). The reader is directed to a recent study by Pandy and Andriacchi<sup>3</sup> for a detailed discussion of muscle function in walking and running.

Second, two simulation environments were used when calculating muscle forces for walking and running. In ideal circumstances, all muscle-force calculations would have been performed in a single simulation environment. However, this was not feasible in the present study because the CMC and NMT methods could not be implemented in the same simulation

environment: the NMT method was available only in Matlab (The Mathworks Inc., Natick, Massachusetts, USA), and the CMC method could be accessed only through OpenSim.<sup>23</sup> Consequently, muscle-force calculations were performed by implementing the SO and NMT methods in Matlab using a musculoskeletal model developed by Anderson and Pandy,<sup>10</sup> whereas the CMC method was implemented in OpenSim using a model developed by Delp et al.<sup>24</sup> While the structure of the skeletal system was identical in both models, the muscle-tendon architecture (i.e. the geometry and mechanical properties of the muscle-tendon units) was different. The OpenSim model<sup>24</sup> was actuated by 92 muscle-tendon units, whereas the model developed by Anderson and Pandy<sup>10</sup> was actuated by 54 muscle-tendon units. Although both models included all the major muscle groups of the lower limb, the OpenSim model divided each muscle group into several distinct sub-regions. Details of the force-producing properties of the muscles included in these models (i.e. maximum



**Figure 4.** Comparison of three sets of muscle-force solutions ( $F^{SO}$ ,  $F^{CMC}$  and  $F^{NMT}$ ) for running at the preferred speed. The horizontal bars indicate the periods of EMG activity recorded for the subject. No EMG data were recorded for ILPSO.

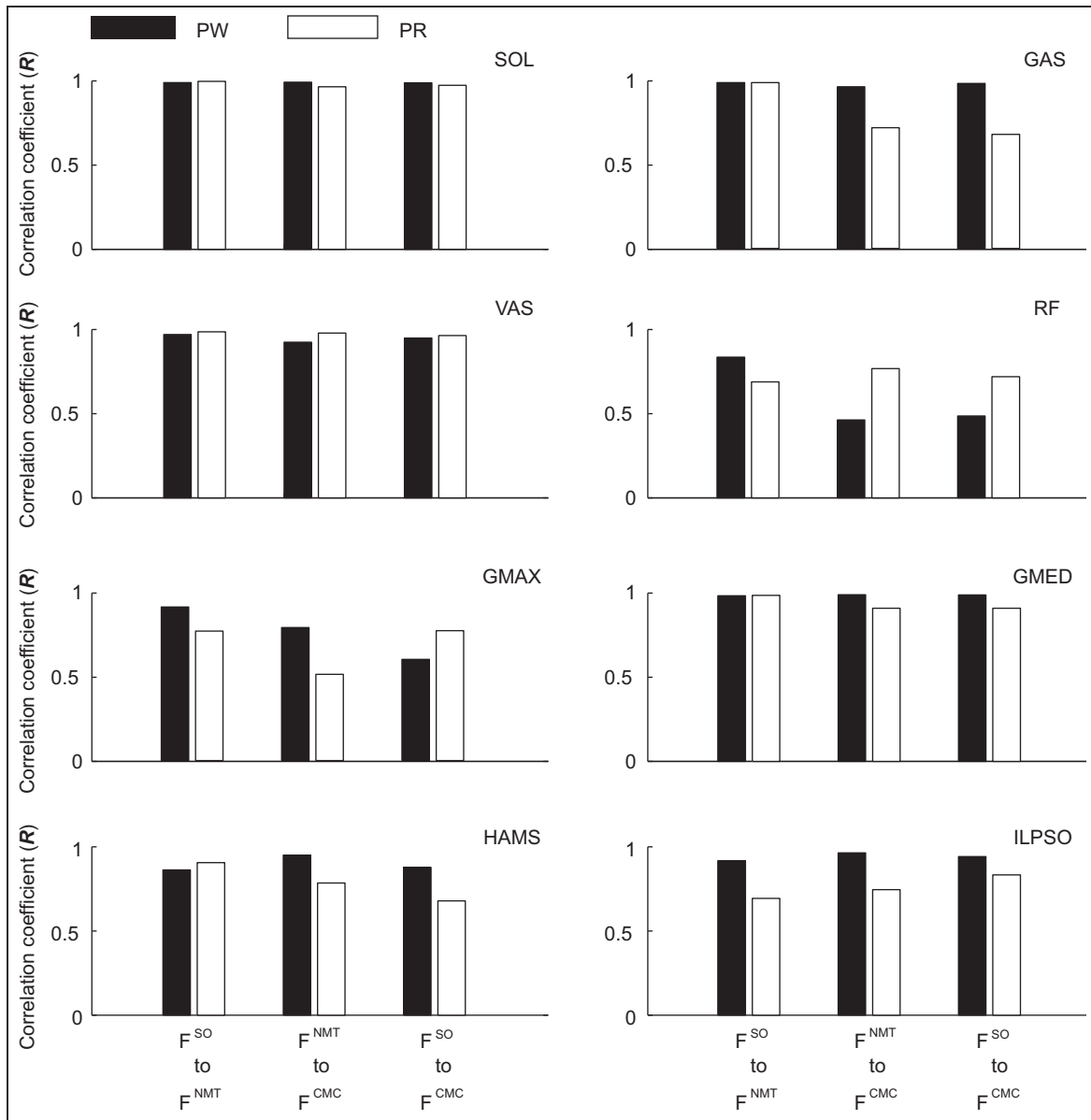
isometric force and the corresponding muscle-fibre length, tendon rest length, maximum shortening velocity, etc.) have been reported previously.<sup>19,23,24</sup> To evaluate the influence of these model differences on the calculated values of muscle forces, we re-solved the walking problem by implementing the SO method in both Matlab and OpenSim. Figure 6 shows that the patterns of muscle loading obtained in Matlab and OpenSim are similar, indicating that muscle-force calculations are not significantly influenced by the simulation environment used. While differences in the magnitudes of the muscle-force estimates are evident in Figure 6, the similarities in the time histories of muscle loading suggest that the two simulation environments will yield a consistent set of predictions for lower-limb muscle function. Finally, the results obtained in the present study may not be applicable to faster speeds of running (i.e. beyond 3.5 m/s) or to ballistic movements such as vertical jumping, where

the influence of muscle activation dynamics may be more pronounced.

Differences in muscle-force estimates obtained from inverse- and forward-dynamics methods can arise from one or more of the following factors:

- the performance criterion assumed;
- errors obtained in tracking biomechanical gait data, for example, joint angular accelerations and joint torques;
- the inclusion of muscle activation dynamics in the formulation of the optimisation problem.

Previous studies have shown that the performance criterion can significantly influence predictions of muscle forces in human movement.<sup>2,4</sup> All three methods implemented in the present study minimised the sum of squares of muscle activations. The main difference was that the SO and CMC methods solved a series of



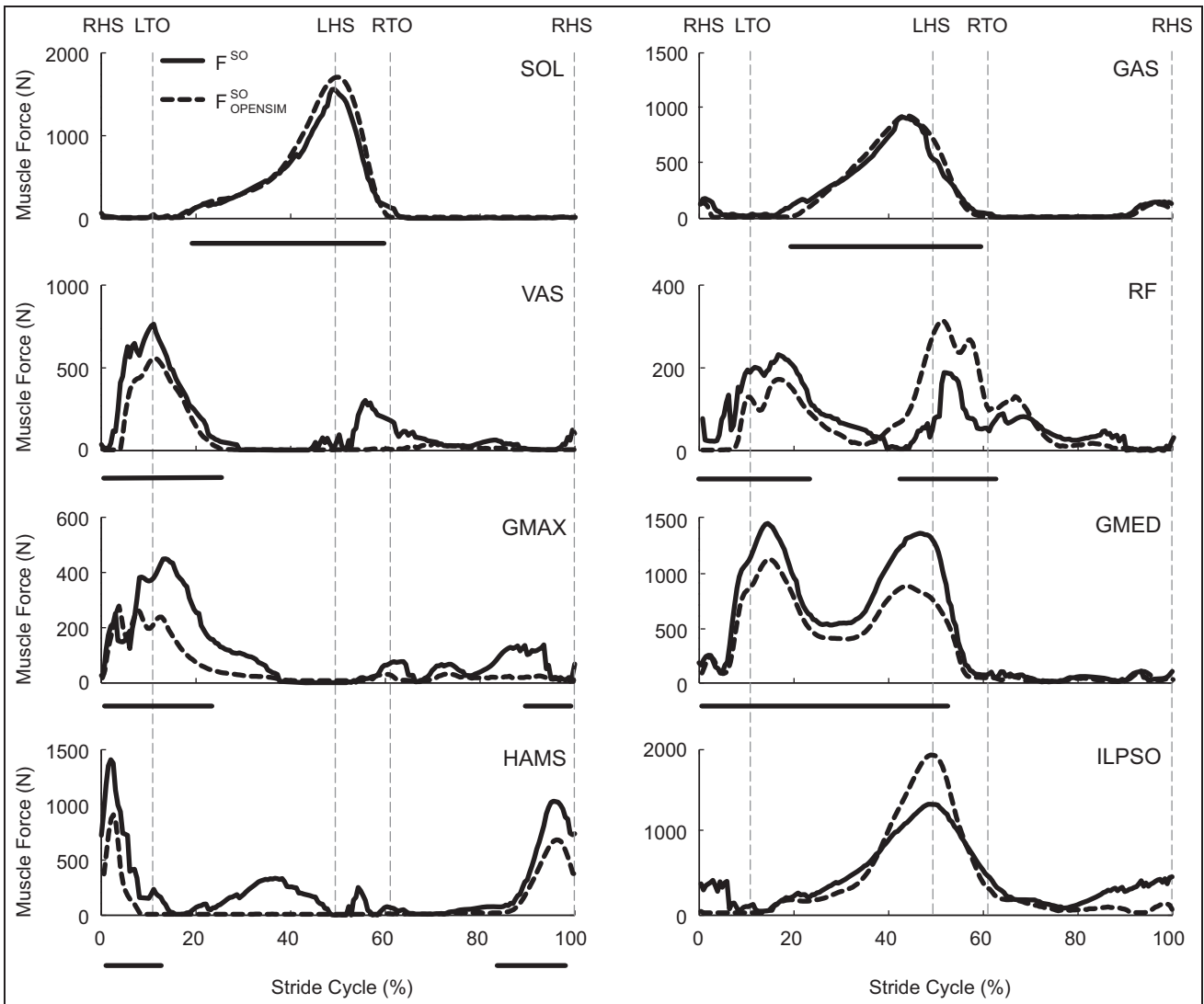
**Figure 5.** The correlations between the three sets of muscle-force solutions:  $F^{SO}$ ,  $F^{CMC}$  and  $F^{NMT}$ . The walking and running results were presented in black and grey bars, respectively.

separate optimisation problems, one at each time instant during the stride cycle, whereas the NMT method solved just one problem over the entire duration of the task (Table 1). Both CMC and NMT reproduced the net joint torques measured for walking and running (Figure 1), suggesting that differences in muscle-force estimates obtained from SO and CMC are due to the influence of muscle activation dynamics, whereas differences obtained from CMC and NMT are due to the influence of a time-dependent performance criterion (see Table 1). The fact that SO and CMC produced similar results suggests that muscle activation dynamics does not have a significant influence on model predictions of muscle forces. It was also found that CMC and NMT produced similar results, which indicates that muscle-force solutions are also not

heavily influenced by the presence of a time-dependent performance criterion. Taken together, these findings suggest that valid estimates of muscle forces in walking and running may be obtained by implementing static optimisation alone.

Accuracy, robustness and efficiency are three major considerations when selecting the most suitable method for calculating muscle forces during human movement, especially when large numbers of subjects are involved. *Accuracy* refers to the ability of a method to produce valid estimates of muscle forces. Because non-invasive measurement of muscle forces is not possible, model predictions of muscle forces are often validated against EMG measurements of muscle activity. While the temporal patterns of muscle forces calculated for both walking and running were consistent with





**Figure 6.** Comparison of two sets of SO solutions ( $F^{SO}$  and  $F^{SO}_{OPENSIM}$ ) for walking at the preferred speed. The curve  $F^{SO}$  is the same as the curve labeled  $F^{SO}$  in Figure 3. The curve  $F^{SO}_{OPENSIM}$  was calculated by implementing the SO method in OpenSim using a model developed by Delp et al.<sup>24</sup> The horizontal bars indicate the periods of EMG activity recorded for the subject. No EMG data were recorded for ILPSO.

measurements of muscle EMG, no data are available to directly validate the magnitudes of the predicted muscle forces.

*Robustness* refers to the ability of a method to produce accurate estimates of muscle forces, even when small changes are introduced into the model and/or the experimental data. Because the process of numerical integration is prone to accumulation of error over time, forward-dynamics methods are inherently less robust than inverse dynamics methods. For example, forward integration of the equations of motion of the neuromusculoskeletal system may progress more slowly, or even terminate, when certain model parameters or user-defined inputs (e.g. integration parameters and tracker weightings) are improperly perturbed. In this study, greater joint-torque tracking errors were evident in the NMT solution (Figures 1 and 2), which resulted directly from the selection of the weighting parameters.

Weighting the importance of minimising joint torque errors more heavily will adversely affect the minimisation of muscular effort; and conversely, a heavier weighting on the importance of minimising muscular effort will adversely affect the minimisation of joint torque errors. An optimisation approach is needed to refine the selection of the weighting parameters in the NMT method.

*Efficiency* refers to the preparation time needed to implement a particular method as well as the computational (CPU) time required to simulate a prescribed task. In this study, static optimisation was approximately five times more efficient than the CMC and NMT methods with respect to actual CPU time. Furthermore, the preparation time for static optimisation was considerably less than that required for the CMC and NMT methods, because the latter two methods required many user-defined inputs to be prescribed

prior to implementation. For example, finding an optimal set of weightings that reduces the overall tracking errors in a stable manner and minimises muscular effort simultaneously is not trivial, especially when dealing with non-linear dynamical systems of high dimension.<sup>8</sup>

Whereas the static optimisation method possesses several advantages in relation to accuracy, robustness and efficiency, it may not always be the most appropriate method for calculating muscle forces in human movement. In particular, it is recommended that caution be taken when static optimisation is used to calculate muscle forces under the following circumstances.

1. In ballistic tasks such as jumping<sup>19</sup> and sprint cycling,<sup>25</sup> muscle activation dynamics act to prevent a muscle from being activated instantaneously in response to a neural excitation signal. Although the results of the present study suggest that muscle activation dynamics may be neglected when calculating muscle forces for walking and slower speeds of running, the delay between muscle excitation and muscle activation may be important in ballistic-type movements. In the absence of muscle activation dynamics, a static optimisation minimisation criterion will favour a muscle with a larger maximum isometric force because of its potential to contribute to the required joint moment. As a result, static optimisation solutions may yield muscle activation patterns that are inconsistent with measured EMG.
2. In tasks that inherently involve a time-dependent performance criterion. For example, the performance criterion of maximum-height jumping can be characterised by the vertical height achieved by the centre of mass.<sup>19</sup> Sprint running is another example where the time-dependant performance criteria may be defined by maximising muscular power generation<sup>26, 27</sup> or vertical ground impulse<sup>28</sup> over the entire stride cycle. Because a time-independent performance criterion cannot accurately model the goal of such tasks, muscle activation patterns predicted by static optimisation may be inconsistent with measured EMG.

To summarise, the results of the present study suggest that muscle activation dynamics and time-dependent performance criteria do not significantly affect calculations of muscle forces obtained for walking and running. Because all three methods (SO, CMC and NMT) produce similar results, the robustness and efficiency of static optimisation make it the most attractive method for estimating muscle forces in human locomotion.

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### References

1. Tsirakos D, Baltzopoulos V and Bartlett R. Inverse optimization: functional and physiological considerations related to the force-sharing problem. *Crit Rev Biomed Engng* 1997; 25: 371–407.
2. Erdemir A, McLean S, Herzog W and Van den Bogert AJ. Model-based estimation of muscle forces exerted during movements. *Clin Biomech (Bristol, Avon)* 2007; 22: 131–154.
3. Pandy MG and Andriacchi TP. Muscle and joint function in human locomotion. *Ann Rev Biomed Engng* 2010; 12: 401–433.
4. Crowninshield RD. Use of optimization techniques to + predict muscle forces. *J Biomech Engng* 1978; 100: 88–92.
5. An KN, Kaufman KR and Chao EYS. Physiological considerations of muscle force through the elbow joint. *J Biomech* 1989; 22: 1249–1256.
6. Anderson FC and Pandy MG. Static and dynamic optimization solutions for gait are practically equivalent. *J Biomech* 2001; 34: 153–161.
7. Pandy MG, Garner BA and Anderson FC. Optimal control of non-ballistic muscular movements: a constraint-based performance criterion for rising from a chair. *Trans ASME, J Biomech Engng* 1995; 117: 15–26.
8. Seth A and Pandy MG. A neuromusculoskeletal tracking method for estimating individual muscle forces in human movement. *J Biomech* 2007; 40: 356–366.
9. Menegaldo LL, De Toledo Fleury, A and Weber HI. A ‘cheap’ optimal control approach to estimate muscle forces in musculoskeletal systems. *J Biomech* 2006; 39: 1787–1795.
10. Anderson FC and Pandy MG. Dynamic optimization of human walking. *Trans ASME, J Biomech Engng* 2001; 123: 381–390.
11. Davy DT and Audu ML. A dynamic optimization technique for predicting muscle forces in the swing phase of gait. *J Biomech* 1987; 20: 187–201.
12. Thelen DG and Anderson FC. Using computed muscle control to generate forward dynamic simulations of human walking from experimental data. *J Biomech* 2006; 39: 1107–1115.
13. Thelen DG, Anderson FC and Delp SL. Generating dynamic simulations of movement using computed muscle control. *J Biomech* 2003; 36: 321–328.
14. Chumanov ES, Heiderscheit BC and Thelen DG. The effect of speed and influence of individual muscles on hamstring mechanics during the swing phase of sprinting. *J Biomech* 2007; 40: 3555–3562.
15. Glitsch U and Baumann W. The three-dimensional determination of internal loads in the lower extremity. *J Biomech* 1997; 30: 1123–1131.
16. Hamner SR, Seth A and Delp SL. Muscle contributions to propulsion and support during running. *J Biomech* 2010; 43: 2709–2716.
17. Schache AG, Kim HJ, Morgan DL and Pandy MG. Hamstring muscle forces prior to and immediately following an acute sprinting-related muscle strain injury. *Gait Posture* 2010; 32: 136–140.

18. Yokozawa T, Fujii N and Ae M. Muscle activities of the lower limb during level and uphill running. *J Biomech* 2007; 40: 3467–3475.
19. Anderson FC and Pandy MG. A dynamic optimization solution for vertical jumping in three dimensions. *Comput Methods Biomech Biomed Engng* 1999; 2: 201–231.
20. Zajac FE. Muscle and tendon: properties, models, scaling, and application to biomechanics and motor control. *Crit Rev Biomed Engng* 1989; 17: 359–411.
21. Lu TW and O'Connor JJ. Bone position estimation from skin marker co-ordinates using global optimisation with joint constraints. *J Biomech* 1999; 32: 129–134.
22. Cappellini G, Ivanenko YP, Poppele RE and Lacquaniti F. Motor patterns in human walking and running. *J Neurophysiol* 2006; 95: 3426–3437.
23. Delp SL, Anderson FC, Arnold AS, et al. OpenSim: open-source software to create and analyze dynamic simulations of movement. *IEEE Trans Biomed Engng* 2007; 54: 1940–1950.
24. Delp SL, Loan JP, Hoy MG, et al. An interactive graphics-based model of the lower extremity to study orthopaedic surgical procedures. *IEEE Trans Biomed Engng* 1990; 37: 757–767.
25. Martin JC, Brown NA, Anderson FC and Spirduso WW. A governing relationship for repetitive muscular contraction. *J Biomech* 2000; 33: 969–974.
26. Cavagna GA, Komarek L and Mazzoleni S. The mechanics of sprint running. *J Physiol* 1971; 217: 709–721.
27. Ward-Smith AJ. A mathematical theory of running, based on the first law of thermodynamics, and its application to the performance of world-class athletes. *J Biomech* 1985; 18: 337–349.
28. Weyand PG, Sternlight DB, Bellizzi MJ and Wright S. Faster top running speeds are achieved with greater ground forces not more rapid leg movements. *J Appl Physiol* 2000; 89: 1991–1999.